Spatial Environmental Economics

Lecture 9: Spatial Inequality

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Roadmap

1. Inequality Across Space and Time
2. A Model of Spatial Inequality
3. Environmental Quality and Urban Inequality

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How to measure inequality?

- Various inequality measures we could use
- All based on looking at income distribution and computing some summary statistic
- Examples:
  - The ratio of the 90th to the 10th percentile income
  - Income share of the bottom 40% of the population
  - Coefficent of variation of income
  - The Gini coefficient

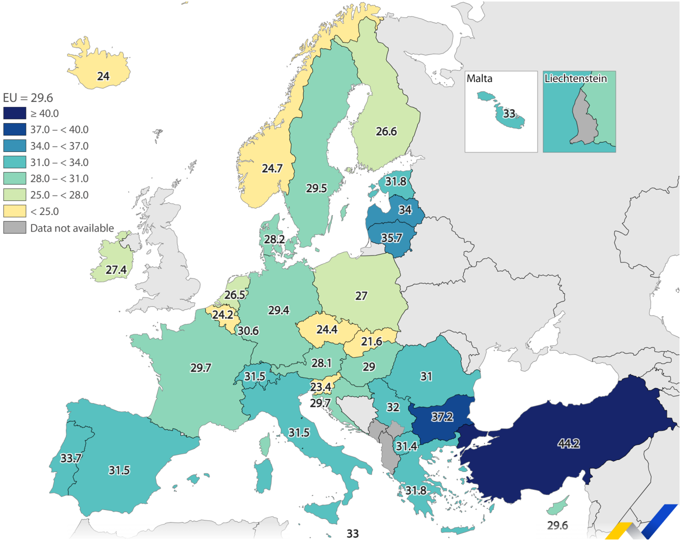
Gini coefficient

- Definition:

$$G = \frac{\sum_i \sum_j |w_i - w_j|}{2N \sum_j w_j}$$

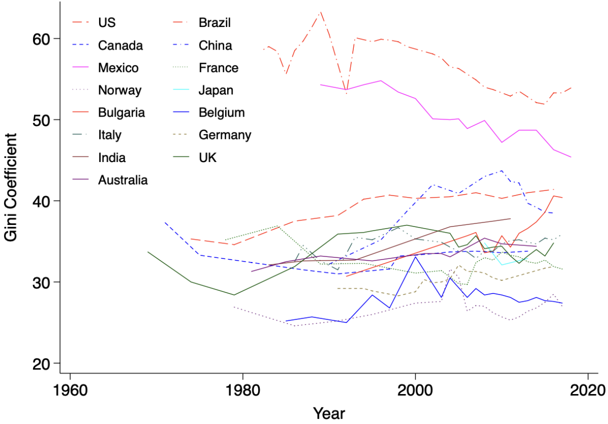
  - w_i is the income of individual i
  - N is the total number of individuals
  - Notice that we normalize by total income so it is about inequality, not levels
- Interpretation:
  - If everyone has the same income, $G = 0$
  - If only 1 person makes all income, then $G \to 1$.
  - If 1% makes 50% of all income the coefficient is at least 24.5% (proof uses the Lorenz curve)

Gini coefficient across European countries



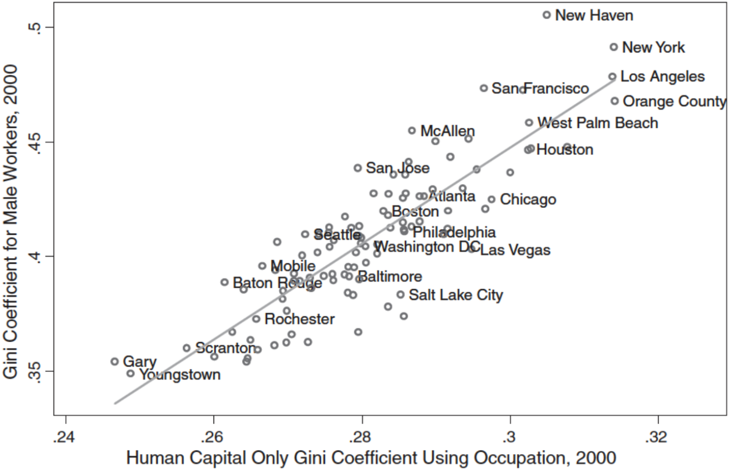
Notes: Gini coefficient for equivalised disposable income per inhabitant, 2023. Source: Eurostat.

Gini coefficient over time for selected countries



Source: Arkolakis (2024). Data from the World Bank.

Gini coefficient correlated with human capital across US cities



Source: Gini coefficients are calculated using the 5 percent Integrated Public Use Microdata Series (IPUMS) for 2000, at usa.ipums.org.

Notes: Glaeser et al. (2009). Gini coefficients for 2000. Human capital computed as years of schooling.

Roadmap

A model of spatial inequality

- In the real world we observe that people with different skills are paid differently
  - E.g. college-educated vs. high-school educated
  - We want a theory (model) that captures that regularity
- How do we model spatial inequality? Incorporate:
  - Two types of labor: L_{Hi} (high education) and L_{Li} (low education)
  - A representative firm i produces using both kinds of labor

Production function

$$Y_i = \left[\left(\bar{A}_{Hi} L_{Hi} \right)^{\frac{\rho-1}{\rho}} + \left(\bar{A}_{Li} L_{Li} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

- CES (constant elasticity of substitution) production function with 2 inputs (L_{Hi}, L_{Li})
- ρ is the elasticity of substitution between the two types of labor
- If $\rho > 1$, the two types of labor are (imperfect) substitutes in production
- $\bar{A}_{Hi}$ and $\bar{A}_{Li}$ are the relative importance (or productivity) of each type of labor
- If $\rho \to \infty$ and $\bar{A}_{Hi} = \bar{A}_{Li} = \bar{A}_i$, we go back to $Y_i = \bar{A}_i \left(L_{Hi} + L_{Li} \right) = \bar{A}_i L_i$ (only total labor matters)

Cost minimization problem

- Firm will choose how much of each type of labor to use to minimize costs
- Formally, a competitive firm solves

$$\min_{L_H, L_L} w_{Hi} L_H + w_{Li} L_L$$

subject to

$$\left[\left(\bar{A}_{Hi} L_{Hi} \right)^{\frac{\rho-1}{\rho}} + \left(\bar{A}_{Li} L_{Li} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \geq Y_i$$

- Specifically, consider the labor allocation choice to produce one unit of output:

$$\left[\left(\bar{A}_{Hi} L_{Hi} \right)^{\frac{\rho-1}{\rho}} + \left(\bar{A}_{Li} L_{Li} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} = 1$$

Optimal hiring

- The first order conditions with respect to the two choices are

$$
\begin{aligned}
w_{Hi} &= \lambda \left(\bar{A}_{Hi}\right)^{\frac{\rho-1}{\rho}} \left(L_{Hi}\right)^{-\frac{1}{\rho}} \\
w_{Li} &= \lambda \left(\bar{A}_{Li}\right)^{\frac{\rho-1}{\rho}} \left(L_{Li}\right)^{-\frac{1}{\rho}}
\end{aligned}
$$

  - λ is the Lagrange multiplier

- Dividing the two we obtain the relative labor demand:

$$
\frac{L_{Hi}}{L_{Li}} = \left(\frac{w_{Li}}{w_{Hi}}\right)^{\rho} \left(\frac{\bar{A}_{Hi}}{\bar{A}_{Li}}\right)^{\rho-1}
$$

  - Use relatively less H if it is relatively more expensive (higher w_{Hi}/w_{Li})
  - Use relatively more H if it is relatively more productive (higher $\bar{A}_{Hi}/\bar{A}_{Li}$)

Equilibrium risk premium

- In equilibrium, demand and supply of labor of each type should equalize
- Demand: optimal hiring decisions imply the following "skill" premium

$$\frac{w_{Hi}}{w_{Li}} = \left(\frac{\bar{A}_{Hi}}{\bar{A}_{Li}}\right)^{\frac{\rho-1}{\rho}} \left(\frac{L_{Hi}}{L_{Li}}\right)^{-\frac{1}{\rho}}$$

  - Intuitively, higher relative supply means lower relative wages
- Supply: how many high and low education people will live in each location i?

$$W_H = \frac{w_{Hi} u_{Hi}}{P_i},\ W_L = \frac{w_{Li} u_{Li}}{P_i} \text{ for all } i$$

  - I.e., spatial equilibrium for each labor type

Implications

- **Skill-biased technical change**
  - If $\rho > 1$, improvements in technology of high-skill to low skills change inequality in the corresponding direction
  - Specifically: if improvement increases $\bar{A}_{Hi}/\bar{A}_{Li}$ it increases inequality
  - Recent tech. progress (e.g. IT) arguably complementary to high-education labor
- **Skill-biased amenities**
  - Note: $u_{Hi} \neq u_{Li}$, amenity taste can differ by income/education
  - Diamond (2016): evidence that amenities that attracted more high-skill people improved faster in US cities during 1980-2000 (retail, transportation infrastructure, crime, environmental, schools)
  - Can result in even larger inequality on well-being than suggested by the wage premium

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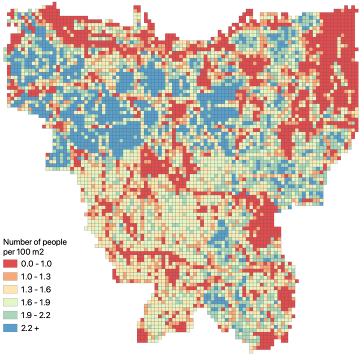
Hsiao (2024): "Sea Level Rise and Urban Inequality"

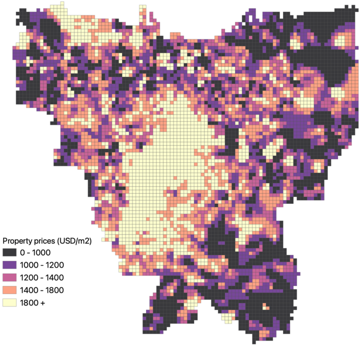
- Questions:
  - Are rich and poor unequally exposed to environmental hazards?
  - Will climate change exacerbate inequality?
- Setting:
  - The flood-prone megacity of Jakarta, Indonesia
  - Rising sea levels due to global warming
- Data:
  - Unit of observation: 300m cells
  - Population shares by educational attainment
  - Home prices
  - Flood frequency

Data

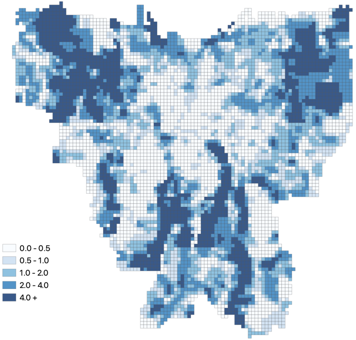
2015 population



2015 home prices



2013-20 flooding frequency



Note: flooding frequency is average months per year with flooding.

Model

- We want a model that (i) uses the data available and (ii) answers the research question
  - Data available: home prices (R_i), flooding days (f_i), populations (L_{gi}) for each cell (i)
  - Define high- and low-wage groups $g \in \{H, L\}$ by post-secondary education
- **Indirect utility** for every location i and wage group g

$$W_{ig} = u_{ig} R_i^{-\beta_g}$$

  - Price elasticities β_g can differ by wage group
- **Amenities** given by

$$u_{ig} = \exp\left(\delta f_i + \bar{u}_{ig}\right) L_{ig}^{-1}$$

- **Spatial equilibrium**: $W_{ig} = W_g$ and $L_g = \sum_i L_{ig}$ for each wage group
- **Housing supply**: each location i has a maximum capacity of $\bar{L}_i$

Equilibrium

- Substituting indirect utility + amenities + spatial equilibrium we obtain

$$\frac{L_{ig}}{L_g} = \frac{\exp\left(\delta f_i + \bar{u}_{ig}\right) R_i^{-\beta_g}}{\sum_\ell \exp\left(\delta f_\ell + \bar{u}_{\ell g}\right) R_\ell^{-\beta_g}} \tag{1}$$

$$W_g = L_g^{-1} \sum_\ell \exp\left(\delta f_\ell + \bar{u}_{\ell g}\right) R_\ell^{-\beta_g}$$

+ housing market clearing requires

$$\bar{L}_i = \sum_g L_{ig}$$

- Equations map exogenous variables $(\bar{L}_i, L_g, f_i, \bar{u}_{ig})$ to endogenous variables (L_{ig}, W_g, R_i)

Estimation: deriving an estimating equation

- Hsiao (2024) estimates the model to recover parameters β_g, δ, and $\bar{u}_{ig}$
- Choosing a reference location $i = 0$, equation (1) implies

$$\frac{L_{ig}}{L_{0g}} = \frac{\exp\left(\delta f_i + \bar{u}_{ig}\right) R_i^{-\beta_g}}{\exp\left(\delta f_0 + \bar{u}_{0g}\right) R_0^{-\beta_g}}$$

- Taking logs we obtain a linear estimating equation

$$\Delta \ln L_{ig} = \beta_g \Delta \ln R_i + \delta \Delta f_i + \Delta \bar{u}_{ig}$$

  - The difference operator Δ is relative to the reference location $i = 0$
  - Recover the amenity (differences) $\Delta \bar{u}_{ig}$ as regression residuals

Estimation: endogeneity

- Identification problem is that prices R_i are correlated with unobserved amenities $\bar{u}_{ig}$
  - The reason is sorting: high-amenity locations attract high-wage individuals that bid up prices
- **Instrument**: use terrain ruggedness as a construction cost shifter
  - As construction must flatten rugged terrain (relevance)
  - The exclusion restriction argument is that Jakarta's modest ruggedness does not impede transportation and thus is less salient to residents
- Assume flooding f_i is uncorrelated with amenities $\bar{u}_{ig}$
  - In practice, coastal areas may enjoy pleasant coastal views despite elevated flood risk
  - Conversely, flood-prone areas may suffer from disinvestment in public amenities
  - Controls help to mitigate this concern: coastal and river distances control for water amenities, elevation captures pleasant views, district fixed effects absorb unobserved differences

Estimation: results

	IV		OLS	
	Estimate	SE	Estimate	SE
Log price, low wages	-2.63	(0.45)	-0.15	(0.04)
Log price, high wages	-1.58	(0.61)	0.28	(0.09)
Flooding	-0.09	(0.04)	-0.05	(0.02)
Observations	10,710			
F-statistic	15.50			

Notes: Each pair of columns is one regression, and each observation is a group-cell with low- and high-wage groups and 300 m cells. The IV specification instruments for log prices with ruggedness. Controls include distance to the coast, distance to the nearest river, elevation, and district fixed effects.

Estimation: interpretation of results

- High prices and severe flooding each reduce residential demand
- $\hat{\delta} = -0.09$ implies that an extra day of flooding reduces population by 9%
- The low-wage group is more price sensitive: $\hat{\beta}^L = -2.36 < \hat{\beta}^H = -1.58$
- OLS estimates exhibit upward bias
  - Because of sorting, locations with high unobserved amenities also have high prices. Individuals may therefore choose these locations despite their high prices
  - Ignoring this correlation leads to the false conclusion that individuals are not price sensitive

Seal level rise (SLR)

- Calculate projected flooding in each cell under SLR: f_i^{SLR}
  1. Consider relative sea level rise of 1, 3, and 5 m for Jakarta using a hydrological model
  2. Identify the corresponding 1.4%, 6.4%, and 20.5% of cells that fall below sea level
  3. Assign the maximum flooding observed in the data—24.5 flood days per year— to these inundated cells
- Given f_i^{SLR}, use the model to solve for the resulting prices R_i^{SLR} and populations L_{ig}^{SLR}
  - Fix wage groups, amenities, and housing supply at current levels

Calculating current and projected flood exposure

- Calculate flood exposure as the average faced by individuals in each group g:

$$F_g = \sum_i \frac{L_{ig}}{L_g} f_i$$

  - Current exposure: calculate directly from the data
  - Future exposure: construct F_g^{SLR} using projected flooding f_i^{SLR} and counterfactual population shares L_{ig}^{SLR}/L_g

- Similarly, calculate price incidence by wage group as

$$R_g = \sum_i \frac{L_{ig}}{L_g} \ln R_i$$

Results: flood exposure

	F_L	F_H	F_L/F_H
Current	0.88	0.62	1.40
Projected			
1m SLR	1.03	0.68	1.52
3m SLR	2.00	0.96	2.09
5m SLR	5.77	2.20	2.62

- Flood level is already high and unequal
- At projected levels, flood exposure increases substantially
- Sea level rise also exacerbates inequality

Results: price incidence

	$R_L - R_H$
Current	-0.10
Projected	
1m SLR	-0.10
3m SLR	-0.13
5m SLR	-0.22

- Sea level rise widens the gap between groups but now to the benefit of low-wage individuals
- Lower prices compensate for higher flood exposure, potentially narrowing the welfare gap
- The change from 3 to 5 m is especially large, as the inundated area expands from 6.4% to 20.5% of cells

Role of sorting

- Evaluate flooding exposure and price incidence with projected flooding and counterfactual prices but each with current choice probabilities instead of counterfactual choice probabilities

$$F_g^{NoSorting} = \sum_i \frac{L_{ig}}{L_g} f_i^{SLR}$$

$$R_g^{NoSorting} = \sum_i \frac{L_{ig}}{L_g} \ln R_i^{SLR}$$

- $F_g^{NoSorting}$ captures the direct impacts of increased flooding
  - Depends only on flood projections without the need to estimate and solve a sorting model
  - But in doing so, it assumes immobility and ignores equilibrium responses to sea level rise

Results: role of sorting

		Flooding			Prices
		F_L	F_H	F_L/F_H	$R_L - R_H$
Current		0.88	0.62	**1.40**	-0.10
Projected	1m SLR	1.03	0.68	**1.52**	-0.10
	3m SLR	2.00	0.96	**2.09**	-0.13
	5m SLR	5.77	2.20	**2.62**	-0.22
Projected, no sorting	1m SLR	1.02	0.73	**1.39**	-0.10
	3m SLR	1.93	1.32	**1.47**	-0.11
	5m SLR	5.47	3.72	**1.47**	-0.15

Notes: The first row computes flood exposure and price incidence from observed data. The second panel solves the model for equilibrium prices and choice probabilities under projected flooding from sea level rise. Each row is one counterfactual. The third panel suppresses the impact of sorting. It computes flood exposure and price incidence with projected flooding and counterfactual prices, but imposes current choice probabilities for each.

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Results: role of sorting

- Inequality is greatly attenuated without sorting
  - Inequality in flood exposure is stable across scenarios
  - Inequality in price incidence is similarly attenuated
- Thus, it is sorting that drives the impact of sea level rise on inequality
  - For SLR of 5 m, sorting reduces high-wage exposure to 2.20 flood days/yr relative to 3.72
  - Also raises low-wage flood exposure to 5.77 flood days relative to 5.47 without sorting

Takeaways from Hsiao (2024)

- Simple model shows that SLR will exacerbate inequality in flood exposure
  - Sorting drives this inequality: high-wage individuals seek out flood-safe areas, bidding up prices and pushing low-wage individuals out
  - For Jakarta, relative sea level rise of 5 m will nearly double inequality
- Policymakers must navigate these distributional effects as SLR reshapes urban landscapes
- Comments:
  - Projections capture the spatially heterogeneous impacts of SLR but are likely underestimates
    - Inundation is certainly worse than 24.5 flood days/yr
    - No adjustment to flooding for cells that fall near sea level but not below
  - At the same time, assume no adaptation via government intervention to reduce damages
    - Hsiao (2025) focuses on this government intervention and its associated challenges

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Appendix

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